Fair Lawn
Public Schools
Fair Lawn, NJ

Pre Calculus Honors

Adopted August

2015

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Developed August 2012

Pre-Calculus Honors is constructed to extend the students knowledge and understanding of algebra and trigonometric functions, and to show how they can be applied to real-life problems. The course will cover a strong foundation of pre-calculus concepts, techniques and applications that will be prepare students for Calculus courses. Students will apply their critical thinking skills in conjunction with their math skills throughout the course.

Pre Calculus Honors

Fair Lawn School District

Committee Credits
Pre Calculus Honors Team

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Pre-Calculus Honors

I. Course Synopsis

In Pre-Calculus Honors, instructional time should focus on five critical areas:
(1) Trigonometric Functions and Applications of Trigonometry, (2) The study of relationships within triangles; (3) Developing an understanding of analytic geometry, specifically Conic sections and Polar Graphs; (4) Furthering the understanding of Algebraic Functions with Real-life Applications; (5) Introduction to Calculus topics including, limits, function decomposition, and derivatives

Throughout the course, mathematical concepts will be taught with an emphasis on enduring understandings, essential questions, real-world application, technology, and cross-curricular interaction.

II. Philosophy & Rationale

The purpose of the math curriculum for Pre-Calculus Honors is to help students develop and enhance mathematical abilities required for Calculus. Students should be able to reason logically and apply mathematical skills to real-world activities. Communicating about and through mathematics will enable students to view mathematics as relevant to their lives and understand it as it connects to other areas. Student should be able to make connections among the different strands of mathematics while feeling confident in using quantitative and spatial information to make decisions. The curriculum will enable students to become independent learners with a desire for lifelong learning. Technology will be infused throughout the curriculum.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

CCSS.MATH.PRACTICE.MP1 - Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous

problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.MATH.PRACTICE.MP2 - Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CCSS.MATH.PRACTICE.MP3 - Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

CCSS.MATH.PRACTICE.MP4 - Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.MATH.PRACTICE.MP5 - Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

CCSS.MATH.PRACTICE.MP6 - Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

CCSS.MATH.PRACTICE.MP7 - Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

CCSS.MATH.PRACTICE.MP8 - Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

III. Scope & Sequence

Unit 1: Trigonometric Functions and Applications of Trigonometry (9 Weeks):

1. Study of Unit Circle
    a. Converting between degrees and radians
    b. Special Angle Relationships
    c. Coterminal Angles and Reference Angles
    d. Trigonometric Ratios and their inverses
2. Graphing Trigonometric Functions
    a. Variations on Sine, Cosine, Tangent and their reciprocal functions
    b. Writing Equations given the graphs of trigonometric functions
3. Trigonometric Equations
    a. Solving Algebraically and graphically
    b. Representing solutions based on different intervals
4. Trigonometric Identities
    a. Establishing Pythagorean identities
    b. Using properties of mathematics to verify trigonometric identities
    c. Sum and Difference Identities
    d. Double Angle Identities
    e. Half Angle Identities
    f. Solving Trigonometric Equations

Unit 2: The study of relationships within triangles (4 weeks):

1. Solving Non-Right Triangles
    a. Converting from DMS to Decimal Degrees
    b. Law of Sines and the Ambiguous Case
    c. The Law of Cosines
    d. Area Formulas including Heron's Formula
    e. Finding the area of quadrilaterals
    f. TRIG STAR NSPS

Unit 3: Developing an understanding of analytic geometry, specifically Conic sections and Polar Graphs (4 weeks):

1. Polar Graphing
    a. Complex Coordinates
    b. Trigonometric Coordinates and Rectangular Coordinates
    c. De Moivre's Theorem for Powers
    d. Argand Diagram
    e. Basic Polar Graphs
    f. Graphing Cardiods and Roses,
2. Conic Sections
    a. Circles, Ellipses, Parabolas, Hyperbolas

b. Graphing
c. Writing Equations

Unit 4: Furthering the understanding of Algebraic Functions with Real-life Applications (15 weeks):

1. Graphing Analysis
    a. Interval Notation
    b. Intervals of Increasing and Decreasing
    c. Maximum and Minimum
    d. Concavity and points of inflection
    e. Domain and Range
    f. Graphically and algebraically
    g. Symmetry of Functions
    h. x-axis, y-axis, origin, y=x, y =-x
    i. Even and Odd Functions
    j. Transformations of functions
    k. Inverse Functions
    l. Rational Functions
        i. Finding Horizontal and Vertical Asymptotes
        ii. Finding x- and y- intercepts.
        iii. Finding slant asymptotes and holes
        iv. Graphing rational functions of all different types
        v. Using end behavior to find limits of rational functions
    m. Piecewise Functions
    n. Absolute Value
    o. Greatest Integer Function
    p. Signum Function
    q. Equations and Inequalities
        i. Radical Equations and Inequalities
        ii. Rational Equations and Inequalities
        iii. Absolute Value Equations and Inequalities
        iv. Polynomial Inequalities
    r. Polynomials
        i. Finding zeroes of polynomials
        ii. Rational Root Theorem
        iii. Intermediate Value Theorem
        iv. Graphing Polynomial Functions
        v. Division of Polynomial Functions
    s. Exponential and Logarithmic Functions
        i. Properties of Exponents
        ii. Rational Exponents
        iii. Solving Equations with Exponents
        iv. Properties of Logarithms

v. Evaluating Logarithms
vi. Solving equations with logarithms as exponents
vii. Graphing Logarithmic and exponential functions
viii. Applications of Exponential and Logarithmic Functions
    1. Compound interest and simple interest
    2. Growth, decay, half-life, double-life

Unit 5: Intro to Calculus (4 weeks):

1. Sequence and Series
    a. Recursive Sequences and Series
    b. Arithmetic Sequences and Series
    c. Geometric Sequences and Series
2. Proofs by Induction
3. Binomial Theorem
4. Review of Factoring
    a. Algebraic Atrocities
5. Limits
    a. Algebraically and Graphically
    b. One Sided
    c. Left and Right
6. Fractions
    a. Composition of Fractions
    b. Decomposition of Fractions
7. Derivatives
    a. Formal Definition
    b. Graphically
    c. Basic Rules

IV. Unit Descriptions

Unit 1: Trigonometric Functions and Applications of Trigonometry

Enduring Understanding

1. Trigonometric functions can be used to describe and quantify relationships.
2. The unit circle can calculate the six trigonometric ratios at any point on the circle.
3. Having an understanding of simple identities can open doors to a whole new set of identities.
4. Identities can be used to calculate exact trigonometric values for any angle using the known special triangles.
5. The solutions to trigonometric equations are many times periodic and have multiple solutions.

Essential Questions

1. How can we use trigonometric functions to describe physical relationships?
2. How can the unit circle be used to calculate the six trigonometric ratios?
3. How can we calculate exact values for angles that are not found using one of the special triangles?
4. How can we use known identities to establish new identities?
5. Why must a trigonometric function's domain be restricted to construct its inverse?
6. How can the inverse trigonometric function be used to solve trigonometric equations?

Learning Objectives

Students will be able to:

1. Convert between radian, degree, and DMS measurements.
2. Find the exact value of the trigonometric functions using a point on the unit circle.
3. Determine the amplitude, period and phase shift of sinusoidal functions.
4. Find a sinusoidal function from a given graph.
5. Establish identities using the Pythagorean Identities and reciprocal trigonometric functions
6. Use the sum and difference identities to find exact values and establish identities.
7. Use the double and half angle formulas to find exact values and establish identities.
8. Find the exact values and approximate of an Inverse Trigonometric Function.
9. Solve equations involving one trigonometric function.
10. Solve trigonometric equations that are quadratic in form.
11. Solve trigonometric equations using identities.

Common Core State Standards

- HSF.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

- HSF.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- HSF.TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
- HSF.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- HSF.TF.B.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- HSF.TF.B.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*
- HSF.TF.C.8 Prove the Pythagorean identity $\sin2(\theta) + \cos2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
- HSF.TF.C.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

<u>Suggested Activities/Modifications</u>

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
    a. Do Now activities
    b. Classwork
    c. Homework
    d. Use of white boards
    e. Unit Test (extended time when needed)
    f. Review Game
    g. Project
    h. Graphic Organizer
    i. Calculator Use
    j. Assistive Technology
2. English Language Learners.
    a. Students may use a bilingual dictionary.
    b. Read written instructions.
    c. Students may be provided with note organizers / study guides to reinforce key topics.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.

3. Special Education/504 Students.
    a. Students may be provided with note organizers / study guides to reinforce key topics.
    b. Extended time on assessments when needed.
    c. Preferred seating to be determined by student and teacher.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
    f. Establish a non-verbal cue to redirect student when not on task.
    g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
    a. Provide enrichment activities to expand upon the curriculum.
    b. Use higher level questioning techniques in class and on assessments.

New Jersey Core Curriculum Standards – Technology and Career Awareness, Exploration, and Preparation

- See Technology & Career Readiness & 21st Century Skills Standards Curriculum Appendix

Career Readiness Practices

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

Career & Technical Education Content Area: 21st Century Life and Careers Standards

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
- 9.3.12.ED.1 - Apply communication skills with students, parents and other groups to enhance learning and a commitment to learning.

Unit 2: The study of relationships within triangles

Enduring Understanding

1. Angles can be measured in multiple formats
2. The ambiguous case occurs when you are given two angles and one side.
3. Missing parts of a triangle can always be found using the appropriate formulas.

Essential Question(s)

1. What is the relationship of the Law of Cosines, Law of Sines, and area formulas to theorems you previously learned?
2. What is the utility of the Law of Sines and the Law of Cosines?
3. How do you apply the law of sines and cosines to quadrilaterals?
4. How can you determine if a triangle has no solution, one solution, or two solutions?

Learning Objectives

Students will be able to:

1. State and apply the laws of Sine and Cosine to solve problems
2. State and apply the area formulas to find the area of non-right triangles.
3. Convert from DMS to decimal degrees.
4. Solve a quadrilateral by multiple applications of the law of sines and cosines.
5. Solve problems one and two on the TRIG STAR competition.

Common Core State Standards

- HSG.SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- HSG.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.
- HSG.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- HSG.SRT.D.9 (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- HSG.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- HSG.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

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Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
    a. Do Now activities
    b. Classwork
    c. Homework
    d. Use of white boards
    e. Unit Test (extended time when needed)
    f. Review Game
    g. Project
    h. Graphic Organizer
    i. Calculator Use
    j. Assistive Technology
2. English Language Learners.
    a. Students may use a bilingual dictionary.
    b. Read written instructions.
    c. Students may be provided with note organizers / study guides to reinforce key topics.

d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.

3. Special Education/504 Students.
    a. Students may be provided with note organizers / study guides to reinforce key topics.
    b. Extended time on assessments when needed.
    c. Preferred seating to be determined by student and teacher.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
    f. Establish a non-verbal cue to redirect student when not on task.
    g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
    a. Provide enrichment activities to expand upon the curriculum.
    b. Use higher level questioning techniques in class and on assessments.

<u>New Jersey Core Curriculum Standards – Technology and Career Awareness, Exploration, and Preparation</u>

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<u>Career & Technical Education Content Area: 21st Century Life and Careers Standards</u>

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
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Unit 3: Developing an understanding of analytic geometry, specifically Conic sections and Polar Graphs

<u>Enduring Understanding</u>

1. Other coordinate systems have their place in mathematics to describe phenomena that is difficult to describe on the Cartesian Coordinate System.
2. Recognize, write, and interpret equations of conic sections
3. Recognize conic sections as useful in applications

<u>Essential Question(s)</u>

1. What are conic sections?

2. What are the formulas that define conic sections such as circles, ellipses and hyperbolas?
3. How are "non-standard form" conic equations converted into standard form formulas?
4. When is a polar system more useful than rectangular?
5. Why is the polar system necessary for finding all solutions for an equation?

Learning Objectives

Students will be able to:

1. Identify and graph conics
2. Write equations of conics from graphs
3. Convert from complex to polar form and vice versa.
4. State and apply DeMoivre's theorem for roots and powers
5. Graph any polar ordered pair or classic curve

Common Core State Standards

- HSG.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- HSG.GPE.A.2 Derive the equation of a parabola given a focus and directrix.
- HSG.GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
- HSN.CN.A.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
- HSN.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- HSN.CN.A.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- HSN.CN.B.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
- HSN.CN.B.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, $(-1 + \sqrt{3}\, i) = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.*

Suggested Activities/Modifications

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Unit 4: Furthering the understanding of Algebraic Functions with Real-life Applications

Enduring Understanding

1. Mathematical models can be used to describe and quantify relationships.
2. Design and formulate mathematical equations to represent graphs of relationships.
3. Patterns and relationships can be represented graphically, numerically, symbolically and verbally
4. You can represent repeated multiplication with an exponential function.
5. You can use logarithms to solve exponential equations and exponential equation to solve logarithmic equations.
6. In order to solve inequalities, you must consider the domain of the function in order to represent the solution set.
7. You can represent all absolute value functions as piecewise functions.

Essential Question(s)

1. How can we use mathematical functions to describe physical relationships?
2. How are patterns of change related to the behavior of functions?
3. Why do we need the logarithm function?
4. What real-world phenomena are modeled by exponential or logarithmic functions?
5. What makes an accurate sketch of a polynomial function?
6. What makes a good window for the graph of a polynomial function?
7. What is the best way to find the zeroes of a polynomial function?
8. What is the importance of piecewise functions?
9. How do the Signum functions and greatest integer function relate to the family of functions and transformation rules?

Learning Objectives

Students will be able to:

1. Determine Domain and Range of functions.
2. Identify the characteristics and shape of various graphs.
3. Determine several ways in which given functions can be used to create new functions.
4. Consider domain when working rational functions.
5. Sketch asymptotes and holes in the graph.
6. Write equations given graphs.
7. Use conjugates to rationalize complex numbers
8. Solve quadratic equations with complex solutions
9. Use the fundamental theorem of algebra to solve polynomial equations
10. Find complex zero of polynomials with degree greater than 2
11. Solve polynomial inequalities and rational inequalities

Common Core State Standards

- HSF.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k

given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- HSF.BF.B.4
- Find inverse functions.
- HSF.BF.B.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- HSF.IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
- HSF.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.*
- HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
- HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
- HSF.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- HSF.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- HSF.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- HSF.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.
- HSA.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- HSA.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

- HSA.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- HSA.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- HSA.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- HSA.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.
- HSA.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- HSA.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- HSA.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- HSA.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- HSA.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve.
- HSA.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

<u>Suggested Activities/Modifications</u>

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
    a. Do Now activities
    b. Classwork
    c. Homework
    d. Use of white boards
    e. Unit Test (extended time when needed)

   f. Review Game
   g. Project
   h. Graphic Organizer
   i. Calculator Use
   j. Assistive Technology
2. English Language Learners.
   a. Students may use a bilingual dictionary.
   b. Read written instructions.
   c. Students may be provided with note organizers / study guides to reinforce key topics.
   d. Provide modified assessments when necessary.
   e. Student may complete assessments in alternate setting when requested.
3. Special Education/504 Students.
   a. Students may be provided with note organizers / study guides to reinforce key topics.
   b. Extended time on assessments when needed.
   c. Preferred seating to be determined by student and teacher.
   d. Provide modified assessments when necessary.
   e. Student may complete assessments in alternate setting when requested.
   f. Establish a non-verbal cue to redirect student when not on task.
   g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
   a. Provide enrichment activities to expand upon the curriculum.
   b. Use higher level questioning techniques in class and on assessments.

<u>New Jersey Core Curriculum Standards – Technology and Career Awareness, Exploration, and Preparation</u>

- See Technology & Career Readiness & 21st Century Skills Standards Curriculum Appendix

<u>Career Readiness Practices</u>

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

<u>Career & Technical Education Content Area: 21st Century Life and Careers Standards</u>

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
- 9.3.12.ED.1 - Apply communication skills with students, parents and other groups to enhance learning and a commitment to learning.

Unit 5: Intro to Calculus

Enduring Understanding

1. Students will understand that a derivative will calculate instantaneous rate of change of a function at a point
2. The chain rule is applied to every derivative rule and it may or may not make an additional contribution to the derivative.
3. The value of a function at a point and its limit value can, but do not have to be the same value.
4. Sequences and series can be expressed in sigma notation and represented using recursion.
5. The binomial theorem will be used as an extension of pascals triangle to find the coefficient of a given term in a polynomial.

Essential Question(s)

1. How can the limit be used to create a tangent line from a given secant line?
2. How can the derivative rules be applied to find derivatives of given functions?
3. How are the value of a function and the value of a limit the same? Different?
4. What is an indeterminate form and what does it mean algebraically and graphically?

Learning Objectives

Students will be able to:

1. Find the derivative using the limit definition
2. Find the derivatives value at a point using the limit definition.
3. Calculate a limit using a table of values.
4. Calculate a limit graphically.
5. Understand how limits relate to vertical and horizontal asymptotes.
6. Decompose fractions with constant and linear coefficients.
7. Decompose fractions with repeated factors.
8. Calculate the coefficient of a given term in a polynomial.
9. Calculate the number of terms, the value of a term, and the sum of series.

Common Core State Standards

- HSF.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*
- HSF.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.*
- HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing,*

positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

- HSF.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
- HSA.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1
- HSA.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- HSA.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Suggested Activities/Modifications

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1. Activities
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Career & Technical Education Content Area: 21st Century Life and Careers Standards

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- 9.3.12.ED.1 - Apply communication skills with students, parents and other groups to enhance learning and a commitment to learning.

V. Course Materials

Textbook

Pre Calculus: Graphical, Numerical, Algebraic. Addison Wesley, 2007

Support books

Advanced Mathematics, A preparation for Calculus, A Coxford, J Payne, 1978 Harcourt
Before Calculus, L. Leithold, 1989 Happer Collins
Advanced Mathematics, Richard Brown, 1994 Houghton Mifflin

Computer and related software

Geometer's Sketchpad, TI-Smartview, PowerPoint, Smart notebook, Slate,
Student Response Software

Calculators

TI N-Spire CX

SAT I and SAT II Review Materials

Barron's, ARCO, College Board Material

Labs using CBR/CBL

Graphing Trigonometric functions using a record player
Movement of the Pendulum
Distance Problems

Smart Board

VI. Assessments

Unit tests and Quizzes
Midterm Exam
Final Exam
Laboratory Activities
Homework
Class Work
Standardized Test Preparation

VII. Cross Curricular Aspects

The Cross-curricular Competencies are four interrelated areas containing understandings, values, skills, and processes that are considered important for learning in all areas of study. These competencies reflect the Common Essential Learnings and are intended to be addressed in each area of study at each grade level.

Developing Thinking
Within their study of mathematics, students must be engaged in personal construction and understanding of mathematical knowledge. This occurs most effectively through student engagement in inquiry and problem solving when they are challenged to think critically and creatively. Moreover, students need to experience mathematics in a variety of contexts – both real world applications and mathematical contexts – in which they consider questions such as "What would happen if ...", "Could we find ...", and "What does this tell us?" Students need to be engaged in the social construction of mathematics to develop an understanding and appreciation of mathematics as a tool that can be used to consider different perspectives, connections, and relationships. Mathematics is a subject that depends upon the effective incorporation of independent work and reflection with interactive contemplation, discussion, and resolution.

Developing Identity and Interdependence
Given an appropriate learning environment in mathematics, students can develop both self-confidence and self-worth. An interactive mathematics classroom in which the ideas, strategies, and abilities of individual students are valued supports the development of personal and mathematical confidence. It also can help students take an active role in defining and maintaining the classroom environment and accepting responsibility for the consequences of their choices, decisions, and actions. A positive learning environment combined with strong pedagogical choices that engage students in learning supports students in behaving respectfully towards themselves and others.

Developing Literacies
Through their mathematical learning experiences, students should be engaged in developing their understanding of the language of mathematics and their ability to use mathematics as a language and representation system. Students should be engaged regularly in exploring a variety of representations for mathematical concepts and expected to communicate in a variety of ways about the mathematics being learned. Important aspects of learning mathematical language are to make sense of mathematics, communicate one's own understandings, and develop strategies to explore what and how others know about mathematics. Moreover, students should be aware of, and able to make, the appropriate use of technology in mathematics and mathematics learning. Encourage students to use a variety of forms of representation (concrete manipulatives; physical movement; oral, written, visual, and other symbolic forms) when exploring mathematical ideas, solving problems, and communicating understandings is important. All too often, symbolic representation is assumed to be the only way to communicate mathematically. The more flexible students are in using a variety of representations to explain and work with the mathematics being learned, the deeper their understanding becomes.

Developing Social Responsibility
As students' progress in their mathematical learning, they need to experience opportunities to share and consider ideas, and resolve conflicts between themselves and others. This requires

that the learning environment constructed by the teacher and students supports respectful, independent, and interdependent behaviors. Every student should feel empowered to help others in developing their understanding, while finding respectful ways to seek help from others. By encouraging students to explore mathematics in social contexts, they become engaged in understanding the situation, concern, or issue, and then in planning for responsible reactions or responses. Mathematics is a subject dependent upon social interaction and, as a result, social construction of ideas. Through the study of mathematics, students learn to become reflective and positively contributing members of their communities. Mathematics also allows for different perspectives and approaches to be considered, assessed for contextual validity, and strengthened.