Fair Lawn
Public Schools
Fair Lawn, NJ

Pre-Calculus CP

Adopted August

2015

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Pre-Calculus CP is constructed to extend the students knowledge and understanding of algebra and trigonometric functions, and to show how they can be applied to real-life problems. The course will cover a strong foundation of pre-calculus concepts, techniques and applications that will be prepare students for Calculus courses. Students will apply their critical thinking skills in conjunction with their math skills throughout the course.

Pre-Calculus CP

Fair Lawn School District

Committee Credits
Pre Calculus CP Team

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Pre-Calculus CP

I. Course Synopsis

In Pre-Calculus CP, instructional time should focus on five critical areas: (1) furthering the understanding of Algebraic Function with Real-life Applications; (2) Studying the properties and applications of Exponential and Logarithmic Functions; (3) developing understanding of Conic sections and where they appear in the Universe; (4) The study of relationships within triangles; (5) Trigonometric Functions.

Throughout the course, mathematical concepts will be taught with an emphasis on enduring understandings, essential questions, real-world application, technology, and cross-curricular interaction.

II. Philosophy & Rationale

The purpose of the math curriculum for Pre-Calculus CP is to help students develop and enhance mathematical abilities required for Calculus. Students should be able to reason logically and apply mathematical skills to real-world activities. Communicating about and through mathematics will enable students to view mathematics as relevant to their lives and understand it as it connects to other areas. Student should be able to make connections among the different strands of mathematics while feeling confident in using quantitative and spatial information to make decisions. The curriculum will enable students to become independent learners with a desire for lifelong learning. Technology will be infused through the curriculum.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

CCSS.MATH.PRACTICE.MP1 - Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if

necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.MATH.PRACTICE.MP2 - Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CCSS.MATH.PRACTICE.MP3 - Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

CCSS.MATH.PRACTICE.MP4 - Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as

writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.MATH.PRACTICE.MP5 - Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

CCSS.MATH.PRACTICE.MP6 - Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

CCSS.MATH.PRACTICE.MP7 - Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in

preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

CCSS.MATH.PRACTICE.MP8 - Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

III. Scope & Sequence

Unit 1: Algebra Review (4 Weeks):

- Linear Equations
  - -Solving
  - -Graphing
- Quadratics Equations
  - -Solving by Factoring
  - -Solving by the Quadratic Formula
  - -Solving by Completing the Square
  - -Solving by Graphing
- Absolute Value Equations
  - -Solving Algebraically
  - -Graphing

Unit 2: Advanced Functions and Transformations (5 weeks):

- Transforming "Mother Functions"
- Domain and Range of All Functions
- Intervals of Increasing and Decreasing
- Finding Extrema
- Linear Functions
- Quadratic Functions
- Absolute Value Functions
- Radical Equations and Functions
  - -Solving Algebraically
  - -Graphing
- Operations on Functions
  - -Adding, Subtracting and Multiplying Functions
  - -Composing Functions
- Rational Functions
  - -Restrictions
  - -Common Factors (holes)
  - -End Behavior
    - -Horizontal Asymptotes
    - -Slant Asymptotes
- Piecewise Functions

Unit 3: Exponential and Logarithms (4 weeks):

- Properties of Exponential and Logarithmic Expression
- Exponential and Logarithmic Functions
  - -Graphing
  - -Domain and Range
  - -Intervals of Increasing and Decreasing
- Word Problems

Unit 4: Conic Sections (4 Weeks):

- Circles
- Ellipses
- Parabolas
- Hyperbolas
  - -Graphing
  - -Competing the Square
  - -Writing Equations

Unit 5: Introduction to Trigonometry (12 weeks):

- Right Triangle Trigonometric Ratios
- Special Right Triangles
- Applications of Right Triangle Trigonometry
  - -Angle of Elevation
  - -Angle of Depression
- Angle Measure
  - -Degrees
  - -Radians
- The Unit Circle
  - -Angles in Standard Position
  - -Co-terminal Angles
  - -Reference Angles
  - -Trigonometric Ratios of Major Angles
  - -Trigonometric Ratios of Minor Angles
    - -Sum and Difference Angle Identities
    - -Double and Half Angle Identities
  - -The Reciprocal Trigonometric Ratios
- Solving Trigonometric Equations
  - -Finding Angles Using Inverse Trigonometry
- Solving Scalene Triangles
  - -Law of Sines
    - -The Ambiguous Case
  - -The Law of Sines
  - -Area Formulas
- Proving Trigonometric Identities
  - -The Pythagorean Identities
  - -Rational Identities
  - -Building Common Denominators
  - -Factoring
  - -Simplification

Unit 6: Trigonometric Functions (4 weeks) :

- -Graphing Trigonometric Functions
  - -Domain
  - -Range
  - -Period
  - -Amplitude
  - -Phase Shifts
  - -Vertical Displacement
- -Graphing the Reciprocal Trigonometric Functions
  - -Domain
  - -Range
- -Building Trigonometric Functions
- -Application of Trigonometric Functions (Word Problems)

IV. Unit Descriptions

Unit 1: Algebra Review

Enduring Understanding

1. Linear Equations are important in understanding simple real wold relationships.
2. Quadratic Equations help model physics and business applications.
3. Understanding the input, output relationship of Linear, Quadratic and Absolute Value equations are the foundation of Functions.
4. The graph of an equation is a set of an infinite amount of possible solutions.
5. Different operations in equations produces identifiable patterns for each graph.

Essential Question(s)

1. How can we distinguish between different types of equations?
2. How do you solve Linear Equations?
3. What methods are used to solve Quadratic Equations and when should you use them?
4. What does the graph of an equation represent?
5. How do you find the initial vale or y - intercept of any graphical equation?

Learning Objectives

Students will be able to:

1. Solve a Linear Equation.
2. Solve a Quadratic Equation.
3. Solve an Absolute Value Equation.
4. Graph Linear, Quadratic and Absolute Value equations in two variables.
5. Find the y - intercept of the graph of any equation with two variables.
6. Find the Roots or Zeros of a Quadratic Equation with two variables.

Common Core State Standards

- HSF.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- HSF.IF.C.7.A Graph linear and quadratic functions and show intercepts, maxima, and minima.
- HSF.IF.C.9 Compare properties of two functions each represented in a different way.
- HSF.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- HSF.LE.A.1.B Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- HSF.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context.

Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
    a. Do Now activities
    b. Classwork
    c. Homework
    d. Use of white boards
    e. Unit Test (extended time when needed)
    f. Review Game
    g. Project
    h. Graphic Organizer
    i. Calculator Use
    j. Assistive Technology
2. English Language Learners.
    a. Students may use a bilingual dictionary.
    b. Read written instructions.
    c. Students may be provided with note organizers / study guides to reinforce key topics.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
3. Special Education/504 Students.
    a. Students may be provided with note organizers / study guides to reinforce key topics.
    b. Extended time on assessments when needed.
    c. Preferred seating to be determined by student and teacher.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
    f. Establish a non-verbal cue to redirect student when not on task.
    g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
    a. Provide enrichment activities to expand upon the curriculum.
    b. Use higher level questioning techniques in class and on assessments.

New Jersey Core Curriculum Standards – Technology and Career Awareness, Exploration, and Preparation

- See Technology & Career Readiness & 21st Century Skills Standards Curriculum Appendix

Career Readiness Practices

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

Career & Technical Education Content Area: 21st Century Life and Careers Standards

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
- 9.3.12.ED.1 - Apply communication skills with students, parents and other groups to enhance learning and a commitment to learning.

Unit 2: Advanced Functions and Transformations

Enduring Understanding

1. Transformations of the parent reciprocal function include stretches, compressions or shrinks, reflections, and horizontal and vertical translations.
2. A rational function is a ratio of polynomial functions. If a rational function is in simplified form and the polynomial in the denominator is not constant, the graph of the rational function features asymototic behavior. It looks quite different from the graphs of either of its polynomial components.
3. You can use much of what you know about adding, subtracting, multiplying, and dividing fractions to add, subtract, multiply, and divide rational functions
4. To solve an equation containing rational expressions, first multiply each side by the least common denominator of the rational expressions. Doing this, however, can introduce extraneous solutions.
5. Solving a square root equation may require that you square each side of the equation, this may introduce extraneous solutions.
6. You can add, subtract, multiply, and divide functions based on how you perform these operations with real numbers. One difference however, is that you must consider the domain of each function.
7. You can combine like radicals using properties of real numbers.

Essential Question(s)

1. Are two quantities inversely proportional if an increase in one corresponds to a decrease in the other?
2. What kinds of asymptotes are possible for a rational function?
3. Are a rational expression and its simplified form equivalent?
4. To simplify the nth root of an expression, what must be true about the expression?
5. When you square each side of an equation, is the resulting equation equivalent to the original?

Learning Objectives

Students will be able to:

1. Identify and describe inverse and direct variation functions.
2. Identify if a rational function has asymptotes, then graph the function.
3. Differentiate between vertical, horizontal, and oblique asymptotes.
4. Define the domains of simplified rational expressions to make them equivalent to the originals.

Common Core State Standards

- HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- HSF.IF.C.7.B Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- HSF.IF.C.7.D (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- HSF.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- HSF.IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

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Unit 3: Exponential and Logarithmic Functions

<u>Enduring Understanding</u>

1. You can represent repeated multiplication with an exponential function.
2. You can use logarithms to solve exponential equations and exponential equation to solve logarithmic equations.

<u>Essential Question(s)</u>

1. How do you model a quantity that changes regularly over time by the same percentage?
2. How are exponents and logarithms related?
3. How are exponential functions and logarithmic functions related?

Learning Objectives

Students will be able to:

1. Graph exponential and logarithmic functions.
2. Use exponential functions to model population growth and decay.
3. Use compound interest formulas to predict financial situations.
4. Transform exponential and logarithmic functions.

Common Core State Standards

- HSF.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- HSF.IF.C.7.E Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- HSF.IF.C.8.B Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$, and classify them as representing exponential growth or decay.

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Career & Technical Education Content Area: 21st Century Life and Careers Standards

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Unit 4: Conic Sections

Enduring Understanding

1. Other coordinate systems have their place in mathematics to describe phenomena that is difficult to describe on the Cartesian Coordinate System.
2. Students will understand the difference between all types of conics and be able to apply the formulas.
3. Each point of a parabola is equidistant from a point called the focus and a line called the directrix.
4. In an x-y relationship, replacing x by x-h and y by y-k (with h>0 and k>0) translates the graph of the relation h units to the right and k units up.
5. An equation of a circle centered at the origin and radius r in the coordinate plane can be represented by the equation $x^2 + y^2 = r^2$.
6. There are four types of curves known as conic sections: parabolas, circles, ellipses, and hyperbolas. Each curve has its own distinct shape and properties.
7. A circle is a set of points a fixed distance from one point. An ellipse “stretches” a circle and is the set of two points that have a total fixed distance from two points.

8. The shape of a hyperbola is guided by asymptotes.
9. A conic section is a curve obtained by intersecting a plane and a double cone.
10. The intersection of a cone and a plane parallel to a line along its side is a parabola.

Essential Question(s)

1. What is the intersection of a cone and a plane parallel to a line along the side of a cone?
2. What is the difference between the algebraic representations of ellipses and hyperbolas?
3. How can we use the center of a circle and a point on the circle to generate the equation of the circle?
4. What are the benefits of having the equation of a circle in center-radius form?
5. Given the equation of a conic section, what key characteristics can be used to determine the type of conic section that is being represented?

Learning Objectives

Students will be able to:

1. Graph all four types of conic sections.
2. Change from General form to Standard form.
3. Identify the type of conic section by the standard form.
4. Write the standard form from the graph of a conic section.

Common Core State Standards

- HSF.IF.C.8.A Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- HSF.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- HSG.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
- HSG.GPE.A.2 Derive the equation of a parabola given a focus and directrix.
- HSG.GPE.A.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Suggested Activities/Modifications

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Unit 5: Introduction to Trigonometry

Enduring Understanding

1. The unit circle can calculate the six trigonometric ratios at any point on the circle.
2. Having an understanding of simple identities can open doors to a whole new set of identities.
3. Identities can be used to calculate exact trigonometric values for any angle using the known special triangles.
4. The solutions to trigonometric equations are many times periodic and have multiple solutions.

Essential Question(s)

1. How can we use trigonometric functions to describe physical relationships?
2. How can the unit circle be used to calculate the six trigonometric ratios?
3. How can we calculate exact values for angles that are not found using one of the special triangles?
4. How can we use known identities to establish new identities?
5. Why must a trigonometric function's domain be restricted to construct its inverse?
6. How can the inverse trigonometric function be used to solve trigonometric equations?
7. If you know the value of $sin(\theta)$, how can you find $cos(\theta), tan(\theta), sec(\theta), csc(\theta)$, and $cot(\theta)$?

Learning Objectives

Students will be able to:

1. Convert between radian and degree measurements.
2. Find the exact value of the trigonometric functions using a point on the unit circle.
3. Determine the amplitude, period and phase shift of sinusoidal functions.
4. Find a sinusoidal function from a given graph.
5. Establish identities using the Pythagorean Identities and reciprocal trigonometric functions
6. Use the sum and difference identities to find exact values.
7. Use the sum and difference identities to prove sine and cosine are phase shifts of each other.
8. Use the double and half angle formulas to find exact values.
9. Use identities to express products as sums and sums as products.
10. Find the exact value of an Inverse Trigonometric Function.
11. Find the approximate value of an Inverse Trigonometric Function using the calculator.
12. Find the exact value of expressions involving inverse trigonometric functions.
13. Solve equations involving one trigonometric function.
14. Solve trigonometric equations that are in quadratic form.
15. Solve trigonometric equations using identities.

Common Core State Standards

- HSG.SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- HSG.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.
- HSG.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- HSF.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- HSF.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- HSF.TF.A.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
- HSF.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F.TF.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F.TF.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F.TF.3: (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
- F.TF.4: (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- F.TF.6: (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- F.TF.7: (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★
- *F.TF.8: Prove the Pythagorean identity $\sin^2(\vartheta) + \cos^2(\vartheta) = 1$ and use it find $\sin(\vartheta)$, $\cos(\vartheta)$, or $\tan(\vartheta)$ given $\sin(\vartheta)$, $\cos(\vartheta)$, or $\tan(\vartheta)$ and the quadrant.*
- *F.TF.9: (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.*

Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
    a. Do Now activities
    b. Classwork
    c. Homework
    d. Use of white boards
    e. Unit Test (extended time when needed)
    f. Review Game
    g. Project
    h. Graphic Organizer
    i. Calculator Use
    j. Assistive Technology
2. English Language Learners.
    a. Students may use a bilingual dictionary.
    b. Read written instructions.
    c. Students may be provided with note organizers / study guides to reinforce key topics.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
3. Special Education/504 Students.
    a. Students may be provided with note organizers / study guides to reinforce key topics.
    b. Extended time on assessments when needed.
    c. Preferred seating to be determined by student and teacher.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
    f. Establish a non-verbal cue to redirect student when not on task.
    g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
    a. Provide enrichment activities to expand upon the curriculum.
    b. Use higher level questioning techniques in class and on assessments.

New Jersey Core Curriculum Standards – Technology and Career Awareness, Exploration, and Preparation

- See Technology & Career Readiness & 21st Century Skills Standards Curriculum Appendix

Career Readiness Practices

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.

- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

Career & Technical Education Content Area: 21st Century Life and Careers Standards

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
- 9.3.12.ED.1 - Apply communication skills with students, parents and other groups to enhance learning and a commitment to learning.

Unit 6: Trigonometric Functions

Enduring Understanding

1. Trigonometric functions can be used to describe and quantify relationships.
2. If a function's domain is restricted, its inverse can be found if it was unable to be found on an entire domain.
3. Periodic behavior is a behavior that repeats over intervals of constant length.
4. The measure of an angle in standard position is the input for two important functions. The outputs are the coordinates (called cosine and sine) of the point of the terminal side of the angle that is one unit from the origin.
5. You can translate periodic functions in the same way that you can translate other functions.

Essential Question(s)

1. How can we use trigonometric functions to describe physical relationships?
2. Why must a trigonometric function's domain be restricted to construct its inverse?
3. How can you model periodic behavior?
4. What function has as its graph a sine curve with amplitude 4, period π, and minimum at the origin?

Learning Objectives

Students will be able to:

1. Determine the amplitude, period and phase shift of sinusoidal functions.
2. Find a sinusoidal function from a given graph.
3. Identify and explore periodic behavior.
4. Graph periodic functions.
5. Write the formulas of trigonometric equations.

Common Core State Standards

- HSF.TF.A.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

- **F.TF.4:** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- **F.TF.6:** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

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Career & Technical Education Content Area: 21st Century Life and Careers Standards

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- 9.3.12.ED.1 - Apply communication skills with students, parents and other groups to enhance learning and a commitment to learning.

V. Course Materials

1. Textbook: Pre Calculus, Graphical, Numerical, Algebraic. Addison Wesley, 2007

2. Support books: Advanced Mathematics, A preparation for Calculus, A Coxford, J Payne, 1978 Harcourt Before Calculus, L. Leithold, 1989 Happer Collins Advanced Mathematics, Richard Brown, 1994 Houghton Mifflin

3. Computer and related software Geometer's Sketchpad TI-Smartview PowerPoint Smart notebook Slate Quizdom Student Response Clickers

4. Calculators TI N-Spire CX

5. SAT I and SAT II review materials Barron's, ARCO, College Board Material

6. HSPA material New Jersey State Department of Education

7. Labs using CBR/CBL Graphing Trigonometric functions using a record player Movement of the Pendulum Distance Problems

8. Smart Board

VI. Assessments

List formative & summative assessments that will be used throughout the course, including, but not limited to, midterms & finals.

1. Unit test and quizzes
2. Midterm exam
3. Final exam
4. Laboratory activities

a) Transformations b) Pendulum to graph sine and cosine c) Ferris Wheel lab

5. Homework
6. Standardized Test Preparation
7. Computer Labs
8. Individual White Boards
9. Exit Tickets
10. Student Response Systems
11. Class Work

VII. Cross Curricular Aspects

Please note any potential opportunities to integrate cross curricular activities into this course.

- *While exploring topics within exponential functions we will discuss how the growth of bacteria can be modeled by an exponential function.*

- *When investigating quadratic functions we will be modeling the flight path of projectiles similarly to physics.*
- *Trigonometric functions involve sinusoidal functions that can model sound waves, radio waves, and periodic motion that is prevalent in a physics course.*