Fair Lawn

Public Schools

Fair Lawn, NJ

Calculus Honors

Adopted August

2015

Revised August 2015
Developed August 2012

The Calculus Honors course has been designed for students who have met the pre-requisite in either Pre-Calculus Honors or Pre-Calculus College Prep. Students may receive college credit through Fairleigh Dickenson University, as this aligns with a college calculus curriculum.

Calculus Honors

Fair Lawn School District

Committee Credits

Jennifer Lubonski, Teacher
Lauren Gimon, Supervisor

Spring 2015

Calculus Honors

I. Course Synopsis

In Calculus Honors, instructional time should focus on three critical areas: (1) Evaluating Limits and Continuity; (2) Derivatives of Functions; (3) Integration and Anti-Differentiation. Throughout the course, mathematical concepts will be taught with an emphasis on enduring understandings, essential questions, real-world application, technology and cross-curricular interaction.

II. Philosophy & Rationale

The purpose of this course is to provide the student with an overview and background in Calculus. It is intended to prepare the student for a college course in Calculus, (and/or higher level mathematics), with a focus on analyzing practical situations relating to Calculus. Students will be given opportunities to develop and apply their analytical skills to problems in mathematical, business and scientific venues. Students will use technology such as the Texas Instruments "TI-Nspire" Calculator on a daily basis.
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

CCSS.MATH.PRACTICE.MP1 - Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically

proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

CCSS.MATH.PRACTICE.MP2 - Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

CCSS.MATH.PRACTICE.MP3 - Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

CCSS.MATH.PRACTICE.MP4 - Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify

important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

CCSS.MATH.PRACTICE.MP5 - Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

CCSS.MATH.PRACTICE.MP6 - Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

CCSS.MATH.PRACTICE.MP7 - Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a

square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

CCSS.MATH.PRACTICE.MP8 - Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

III. Scope & Sequence

Unit 1: Function Analysis (2 Weeks)

Graphing Functions
Piecewise Functions
Domain and Range
Composition of Functions
Review of Trigonometry

Unit 2: Limits and Their Properties (4 Weeks):

Graphical Introduction to Limits
Techniques for Evaluating
Limits to Infinity
Continuity by Definition

Unit 3: Differentiation (10 Weeks):

Limit Definition of Derivative
Rules for Differentiation – Power, Product, Quotient
Tangent and Normal Lines
Velocity, Acceleration and Other Rates of Change
Derivatives of Trig Functions
Chain Rule
Implicit Differentiation
Related Rates
Derivatives of Power, Logarithmic and Inverse Trig Functions

Unit 4: Applications of Derivatives (6 Weeks):

First Derivative Test
Extrema on a Closed Interval
Mean –Value Theorem
Second Derivative Test
Curve Sketching
Optimization Problems

Unit 5: Integration (5 Weeks):

Approximation of Area Under a Curve
Definite Integrals
Indefinite Integrals
Integration of Trig Functions
Integration by U-Substitution
Fundamental Theorem of Calculus

Unit 6: Mathematical Modeling and Applications of Integration (4 Weeks):

Differential Equations

Slope Fields

Area Under a Curve and Between Two Curves

Volume by Curve Rotation

IV. Unit Descriptions

Unit 1: Function Analysis

Enduring Understanding

1. Patterns, functions, and relationships can be represented graphically, numerically, symbolically or verbally.
2. The function and relationship concepts are fundamental ideas in mathematics and understanding that patterns of change relate to the behavior of functions.
3. Students can translate readily between algebra, geometry and trigonometry.
4. Analysis of the critical elements of functions is essential to calculus.

Essential Question(s)

1. How are functions used in problem solving when modeling real world situations?
2. How do domain and range relate to the graph of a function?
3. How do the properties and graphs of trigonometric functions differ from those of traditional functions?

Learning Objectives

Students will be able to

1. Graph functions, piecewise functions and trigonometric functions.
2. Identify the domain and range of a given function.
3. Evaluate functions and composite functions.
4. Determine exact values of trigonometric functions.

Common Core State Standards

- **A.SSE.A.1.B:** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it.
- **A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.B.3.A:** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A.APR.D.6:** Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + ^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

- **A.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.B.4:** Solve quadratic equations in one variable.
- **A.REI.B.4.A:** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **A.REI.B.4.B:** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **F.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
- **F.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.C.7.A:** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F.IF.C.7.B:** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F.IF.C.7.E:** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

- **F.BF.A.1.C:** Compose functions.
- **F.BF.B.3:** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- **F.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Suggested Activities/Modifications

Below is a list of suggested activities, modifications, accommodations, and enrichment opportunities. This includes, but is not limited to,:

1. Activities
    a. Do Now activities
    b. Classwork
    c. Homework
    d. Use of white boards
    e. Unit Test (extended time when needed)
    f. Review Game
    g. Project
    h. Graphic Organizer
    i. Calculator Use
    j. Assistive Technology
2. English Language Learners.
    a. Students may use a bilingual dictionary.
    b. Read written instructions.
    c. Students may be provided with note organizers / study guides to reinforce key topics.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
3. Special Education/504 Students.
    a. Students may be provided with note organizers / study guides to reinforce key topics.
    b. Extended time on assessments when needed.

c. Preferred seating to be determined by student and teacher.
d. Provide modified assessments when necessary.
e. Student may complete assessments in alternate setting when requested.
f. Establish a non-verbal cue to redirect student when not on task.
g. Maintain strong teacher / parent communication.

4. Gifted and Talented Students.
   a. Provide enrichment activities to expand upon the curriculum.
   b. Use higher level questioning techniques in class and on assessments.

New Jersey Core Curriculum Standards - Technology

- 8.1.12.A.3: Participate in online courses, learning communities, social networks, or virtual worlds and recognize them as resources for lifelong learning.

Career Readiness Practices

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

9.2 Career Awareness, Exploration, and Preparation Content Area: 21st Century Life and Careers Strand C: Career Preparation

- 9.2.12.C.1 Review career goals and determine steps necessary for attainment.
- 9.2.12.C.2 Modify Personalized Student Learning Plans to support declared career goals.
- 9.2.12.C.3 Identify transferable career skills and design alternate career plans.

Career & Technical Education Content Area: 21st Century Life and Careers Standards

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
- 9.3.ST-SM.4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.

Unit 2: Limits and Their Properties

Enduring Understanding

1. Algebra, trigonometry and logarithms are essential tools for the study of calculus.
2. The concept of a limit is one of the foundations of calculus.
3. Continuous and discontinuous functions model real-life phenomena.

Essential Question(s)

1. How can you evaluate a limit at a value that is not in the domain of the function?
2. How do limits relate to end behavior of a function?
3. How can we alter a discontinuous function so that it is continuous?
4. What are the graphical, numerical, and analytical ways to determine a limit of a function at a given value and its continuity at that point?
5. What are the features of continuous functions? What are the characteristics of a discontinuous function?
6. What role do limits play as a foundation for the calculus?

Learning Objectives

Students will be able to

1. Differentiate between a limit and a function value, graphically and algebraically
2. Evaluate a limit using various algebraic methods
3. Understand a limit as x approaches infinity
4. Determine if a function is continuous
5. Explain how limits relate to the continuity of a function

Common Core State Standards

- **A.SSE.A.1.B:** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it.
- **A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.B.3.A:** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A.APR.D.6:** Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A.REI.D.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **F.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

- **F.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.C.7.A:** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F.IF.C.7.B:** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F.IF.C.7.D:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- **F.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F.IF.C.8.A:** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- **F.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **F.LE.A.3:** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **F.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

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    a. Students may use a bilingual dictionary.
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    c. Students may be provided with note organizers / study guides to reinforce key topics.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
3. Special Education/504 Students.
    a. Students may be provided with note organizers / study guides to reinforce key topics.
    b. Extended time on assessments when needed.
    c. Preferred seating to be determined by student and teacher.
    d. Provide modified assessments when necessary.
    e. Student may complete assessments in alternate setting when requested.
    f. Establish a non-verbal cue to redirect student when not on task.
    g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
    a. Provide enrichment activities to expand upon the curriculum.
    b. Use higher level questioning techniques in class and on assessments.

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- 8.1.12.A.3: Participate in online courses, learning communities, social networks, or virtual worlds and recognize them as resources for lifelong learning.

<u>Career Readiness Practices</u>

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- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

9.2 Career Awareness, Exploration, and Preparation Content Area: 21st Century Life and Careers
Strand C: Career Preparation

- 9.2.12.C.1 Review career goals and determine steps necessary for attainment.
- 9.2.12.C.2 Modify Personalized Student Learning Plans to support declared career goals.
- 9.2.12.C.3 Identify transferable career skills and design alternate career plans.

Career & Technical Education Content Area: 21st Century Life and Careers Standards

- 9.3.ST.2 Use technology to acquire, manipulate, analyze and report data.
- 9.3.ST-SM.4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.

Unit 3: Differentiation

Enduring Understanding

1. The slope of a line in algebra is the average rate of change while the slope of the tangent to a curve at a point in calculus is the instantaneous rate of change (the derivative of a function).
2. There is a defined relationship between the function f, and the first and second derivatives of function f.
3. Derivatives can be applied to numerous real-world applications, including velocity, acceleration and rate of population and financial growth.

Essential Question(s)

1. How does the concept of a limit lead to a derivative?
2. How are derivatives used to analyze the behavior of a function?
3. What are the conditions for differentiability?
4. What are the various rules for differentiation and how are they applied?
5. How does the derivative relate to velocity, acceleration and other real-world applications?
6. Why is implicit differentiation an important tool in finding derivatives?
7. How is calculus used to solve related rate problems?
8. What role do derivatives play in practical applications of calculus?

Learning Objectives

Students will be able to

1. Discover the power rule of derivatives by using the limit definition

2. Analyze a function to apply the appropriate derivative rules: product, quotient, chain, power, implicit
3. Group a function and its derivative
4. Write the equations for lines tangent and normal lines
5. Identify when a function is differentiable
6. Apply the rules of derivatives to velocity, acceleration and other word problems
7. Discover the chain rule and be able to apply it where necessary
8. Extend the rules of derivatives to the trigonometric, logarithmic and power functions
9. Categorize differentiation of functions vs. non-functions
10. Illustrate related rates problems using calculus methods

Common Core State Standards

- **N.Q.A.1:** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.Q.A.2:** Define appropriate quantities for the purpose of descriptive modeling.
- **N.Q.A.3:** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- **N.VM.A.3:** Solve problems involving velocity and other quantities that can be represented by vectors.
- **A.SSE.A.1.B:** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it.
- **A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.B.3.A:** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A.APR.D.6:** Rewrite simple rational expressions in different forms; write $^{a(x)}/_{b(x)}$ in the form $q(x) + ^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A.CED.A.1:** Create equations and inequalities in one variable and use them to solve problems.

- **A.CED.A.3:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- **A.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.B.4:** Solve quadratic equations in one variable.
- **A.REI.B.4.A:** Use the method of completing the square to transform any quadratic equation in $x$into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **A.REI.B.4.B:** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **A.REI.D.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **F.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F.IF.B.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph
- **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.C.7.A:** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F.IF.C.7.B:** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F.IF.C.7.E:** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

- **F.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **F.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
- **HSG.MG.A.1:** Use geometric shapes, their measures, and their properties to describe objects.
- **HSG.MG.A.2:** Apply concepts of density based on area and volume in modeling situations.
- **HSG.MG.A.3:** Apply geometric methods to solve design problems.

<u>Suggested Activities/Modifications</u>

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    c. Homework
    d. Use of white boards
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    f. Review Game
    g. Project
    h. Graphic Organizer
    i. Calculator Use
    j. Assistive Technology
2. English Language Learners.
    a. Students may use a bilingual dictionary.
    b. Read written instructions.
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    g. Maintain strong teacher / parent communication.
4. Gifted and Talented Students.
    a. Provide enrichment activities to expand upon the curriculum.
    b. Use higher level questioning techniques in class and on assessments.

New Jersey Core Curriculum Standards - Technology

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Career Readiness Practices

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- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP11. Use technology to enhance productivity.

9.2 Career Awareness, Exploration, and Preparation Content Area: 21st Century Life and Careers Strand C: Career Preparation

- 9.2.12.C.1 Review career goals and determine steps necessary for attainment.
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Career & Technical Education Content Area: 21st Century Life and Careers Standards

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Unit 4: Applications of Derivatives

Enduring Understanding

1. The first and second derivative test can be applied in order to sketch a graph of a function. This will work in conjunction with student's prior knowledge of graphing functions.
2. The mean-value theorem applicable to all continuous, differentiable functions.
3. Applying the rules of derivatives to mathematical models, students can solve optimization word problems.

Essential Question(s)

1. How does one use the first and second derivative test to sketch a function?
2. What is a practical application of the mean-value theorem?
3. How is the derivative used to optimize quantities?

Learning Objectives

Students will be able to

1. Demonstrate knowledge of the rules of derivatives to be able to use the first and second derivative test
2. Interpret the first and second derivative test in order to sketch a graph of a function
3. Understand the Mean-Value Theorem and be able to apply it to traditional numeric problems as well as real-world word problems
4. Construct mathematical models to solve optimization word problems, using calculus methods.

Common Core State Standards

- **N.Q.A.1:** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.Q.A.2:** Define appropriate quantities for the purpose of descriptive modeling.
- **N.Q.A.3:** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- **A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.B.3.A:** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3:** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A.CED.A.1:** Create equations and inequalities in one variable and use them to solve problems.

- **A.CED.A.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.CED.A.3:** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
- **A.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.B.4:** Solve quadratic equations in one variable.
- **A.REI.B.4.A:** Use the method of completing the square to transform any quadratic equation in $x$into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **A.REI.B.4.B:** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **F.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
- **F.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
- **F.IF.B.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph
- **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.C.7.A:** Graph linear and quadratic functions and show intercepts, maxima, and minima.

- **F.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F.LE.A.4:** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- **F.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
- **HSG.MG.A.1:** Use geometric shapes, their measures, and their properties to describe objects.
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Suggested Activities/Modifications

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4. Gifted and Talented Students.
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9.2 Career Awareness, Exploration, and Preparation Content Area: 21st Century Life and Careers Strand C: Career Preparation

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Career & Technical Education Content Area: 21st Century Life and Careers Standards

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Unit 5: Integration

Enduring Understanding

1. Integration is the opposite operation to differentiation (similar to multiplication and division.)
2. There is a subtle, yet essential, difference between a definite and indefinite integral.
3. Integral calculus is the study of the area between the curve and the x-axis.
4. The Fundamental Theorem of Calculus connects differential and integral calculus.
5. The Riemann Sum is used to estimate the area between the curve and the x-axis, while the integral gives us the exact value.

Essential Question(s)

1. How can the definite integral be applied to determining the area under a curve?
2. How are integrals used to measure changing quantities?
3. How does the Fundamental Theorem of Calculus connect integral and differential calculus?
4. What are some alternative techniques of integration?

Learning Objectives

Students will be able to

1. Use the rules of integration to evaluate an indefinite integral
2. Apply the Fundamental Theorem of Calculus to compute a definite integral
3. Understand the process of U-substitution for integrals as the reverse of the chain rule for derivatives

Common Core State Standards

- **N.CN.C.9:** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- **A.SSE.A.1.B:** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it.
- **A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.C.7.A:** Graph linear and quadratic functions and show intercepts, maxima, and minima.

- **F.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
- **F.TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Suggested Activities/Modifications

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Unit 6: Mathematical Modeling and Applications of Integration

Enduring Understanding

1. A slope field is a graphic way of representing differential equations.
2. The definite integral can be used to find exact area or volume of revolution.
3. Integrals can be used to solve a variety of problems related to velocity, acceleration and distance travelled.

Essential Question(s)

1. How do differential equations describe rates of change?
2. What is the relationship between slope fields and differential equations?
3. How are the areas of bounded regions and volume of solids evaluated?
4. How can one estimate and calculate the area of the bounded region between two curves?

5. How can the definite integral be used to calculate the volume of a solid with a known cross section?

Learning Objectives

Students will be able to

1. Construct a slope field given a differential equation
2. Solve a differential equation for both its general and particular solution
3. Apply rules of definite integrals to calculate area and volume

Common Core State Standards

- **A.SSE.A.1.B:** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it.
- **A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.CED.A.1:** Create equations and inequalities in one variable and use them to solve problems.
- **A.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
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- **A.REI.B.4.A:** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **A.REI.B.4.B:** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **A.REI.C.6:** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
- **A.REI.C.7:** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.
- **F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- **F.IF.C.8:** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
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V. Course Materials:

Including but not limited to

- Textbook Calculus: Graphical, Numerical, Algebraic
  Publisher: Prentice Hall
  Copyright: 2007
- Ti-Nspire Calculator
- Google Drive and Applications
- Interactive Student Response Software
- Individual Student White Boards

VI. Assessments

Assessments included but not limited to:

- Quizzes
- Tests
- Notebook Assessments
- Do Now Exercises
- In Class Worksheets
- Assignments Completed in Pairs
- Homework
- Midterm Exam and Final Exam
- Final Project

VII. Cross Curricular Aspects

- Calculator Use (Science)
- Projectile Motion Word Problems (Science)
- Related Rates (Science)
- Optimization (Science and Business)
- Slope Field (Art)
- Final Project (Fine Art or Art)