Fair Lawn

Public Schools

Fair Lawn, NJ

Advanced Placement

Calculus BC

Adopted August

2017

Revised August 2017
Developed August 2012

The Advanced Placement Calculus BC course has been designed for students who have met the pre-requisites in previous mathematics courses. Students will take the Advanced Placement Exam administered by the College Board, which will give them the opportunity to earn college credit.

AP Calculus BC

Fair Lawn School District

Committee Credits
Advanced Placement Calculus BC Team

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June 2015

AP Calculus BC

I. Course Synopsis

AP courses in calculus consist of a full high school academic year of work and are comparable to calculus courses in colleges and universities. It is expected that students who take an AP course in calculus will seek college credit, college placement or both from institutions of higher learning. Calculus BC is primarily concerned with developing the students' understanding of the concepts of calculus and providing experience with its methods and applications.
(Source: AP College Board)

II. Philosophy & Rationale

AP Calculus BC focuses on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multi-representational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important. Technology is used regularly to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the course becomes cohesive rather than a collection of unrelated topics.

Broad concepts and widely applicable methods are emphasized. The focus of the course is neither manipulation nor memorization of an extensive taxonomy of functions, curves, theorems or problem types. Thus, although facility with manipulation and computational competence are important outcome, they are not the core of the course. Technology should be used regularly to reinforce relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results.
(Source: AP College Board)

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see *mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*

MATH.PRACTICE.MP1 - Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MATH.PRACTICE.MP2 - Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MATH.PRACTICE.MP3 - Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning

from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

MATH.PRACTICE.MP4 - Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MATH.PRACTICE.MP5 - Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MATH.PRACTICE.MP6 - Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They

are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MATH.PRACTICE.MP7 - Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MATH.PRACTICE.MP8 - Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

THE MATHEMATICAL PRACTICES FOR AP CALCULUS

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. They are drawn from the rich work in the National Council of Teachers of Mathematics (NCTM). Process Standards and the Association of American Colleges and Universities (AAC&U) Quantitative Literacy VALUE Rubric. Embedding these practices in the study of calculus enable students to establish mathematical lines of reasoning and use them to apply mathematical concepts and tools to solve problems. The Mathematical Practices for AP Calculus are not intended to be viewed as discrete items that can be checked off a list; rather, they are highly interrelated tools that should be utilized frequently and in diverse contexts.
(Source: College Board)

MPAC 1 - Reasoning with definitions and theorems.

- Use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results
- Confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem.
- Apply definitions and theorems in the process of solving a problem.
- Develop conjectures based on exploration with technology.
- Produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

MPAC 2 - Connecting concepts.

- Relate the concept of a limit to all aspects of calculus.
- Use the connection between concepts (e.g. rate of change and accumulation) or processes (e.g. differentiation and its inverse process, antidifferentiation) to solve problems.
- Connect concepts to their visual representations with and without technology.
- Identify a common underlying structure in problems involving different contextual situations.

MPAC 3 - Implementing algebraic/computational process.

- Select appropriate mathematical strategies.
- Sequence algebraic/computational procedures logically.
- Complete algebraic/computational processes correctly.
- Apply technology strategically to solve problems.
- Attend to precision graphically, numerically, analytically, and verbally and specify units of measure.
- Connect the results of algebraic/computational process to the question asked.

MPAC 4 - Connecting multiple representations.

- Associate tables, graphs and symbolic representations of functions.
- Develop concepts using graphical, symbolical, or numerical representations with and without technology.

- Identify how mathematical characteristics of functions are related in different representations.
- Extract and interpret mathematical content from any presentation of a function (e.g. utilize information from a table of values.)
- Construct one representational form from another (e.g. a table from a graph, etc.)
- Consider multiple representations of a function to select or construct a useful representation for solving a problem

MPAC 5 - Building notational fluency.
- Know and use a variety of notations
- Connect notation to definitions (e.g. relating the notation for the definite integral to that of the limit of a Riemann sums).
- Connect notation to different representations (graphical, numerical, analytical, and verbal).
- Assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6 - Communicating.
- Clearly present methods, reasoning, justifications, and conclusions.
- Use accurate and precise language and notation.
- Explain the meaning of expressions, notation, and results in terms of a context (inc. units).
- Explain the connections among concepts.
- Critically interpret and accurately report information provided by technology.
- Analyze, evaluate, and compare the reasoning of others.

III. Scope & Sequence

Note: AP Calculus BC meeting 7 periods a week.

Unit 1: FUNCTIONS, GRAPHS, LIMITS AND CONTINUITY (10 periods)

Analysis of graphs
Limits of functions (including one-sided limits)
Asymptotic and unbounded behavior
Continuity as a property of functions

Unit 2: DERIVATIVES AND APPLICATIONS (35 periods)

Concept of a derivative
Derivative of a point
Derivative as a function
Higher order derivatives
Applications of derivatives
Computation of derivatives

Unit 3: INTEGRALS AND THE FUNDAMENTAL THEOREM OF CALCULUS (72 periods)

Interpretation and properties of definite integrals
Applications of integrals
Fundamental Theorems of Calculus
Techniques of antidifferentiation
Applications of antidifferentiation
Numerical approximations to definite integrals

Unit 4: PLANE CURVES, PARAMETRIC EQUATIONS, AND POLAR CURVES (10 periods)

The analysis of planar curves includes those in parametric, polar, and vector forms
Derivatives and applications
Antidifferentiation and applications
Integrals and applications

Unit 5: POLYNOMIAL APPROXIMATIONS AND SERIES (32 periods)

Concept of series
Series of constants
Taylor series

REVIEW FOR AP EXAM (21 periods)

POST EXAM (20 periods)

Additional Topics (not tested on AP Calculus BC Exam)
Final Project

IV. Unit Descriptions

UNIT 1: FUNCTIONS, GRAPHS AND LIMITS

Enduring Understanding

Students will understand that...

1. The concept of a limit can be used to understand the behavior of functions.
2. Limits can be estimated from graphs, tables of values, or using algebraic manipulation.
3. Continuity is a key property of functions that is defined using limits.
4. Functions can be represented graphically, numerically, symbolically, and verbally.

Essential Questions

1. What are the properties of linear, quadratic, exponential, parametric, and logarithmic equations?
2. How can the limit of a function exist at values of *x* not included in the domain of the function?
3. How do limits help describe the end behavior of a function?
4. What is the difference between average and instantaneous rates of change?
5. How can graphs and tables be used to determine limits?
6. Can discontinuous functions be redefined so as to make them continuous?
7. What is the connection between the limit of a function and its continuity at a given point?
8. What is the difference between continuity at a point and a continuous function?

Learning Objectives

Students will be able to....

1. Students will be able to perform advanced manipulation and analysis to functions in order to derive limits.
2. Express limits symbolically using correct notation, as well as interpret limits expressed symbolically.
   - Given a function f, the limt of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$.
   - The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits.
   - A limit might not exist for some functions at particular values of $f(x)$. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.
3. Estimate limits of functions.
   - Numerical and graphical information can be used to estimate limits.
4. Determine limits of functions.
   - Limits of sums, differences, products, quotients, and composite functions can be found using the basic theorems of limits and algebraic rules

- The limit of a function may be found by using algebraic manipulation, alternate forms of trigonometric functions (trig identities), or the squeeze theorem.
- Limits of the indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$ may be evaluated using L'Hospital's Rule.

5. Deduce and interpret behavior of functions using limits.
   - Asymptotic and unbounded behavior of functions can be explained using limits.
   - Relative magnitudes of functions and their rates of change can be compared using limits.
6. Analyze functions for intervals of continuity or points of discontinuity.
   - A function f is continuous at $x = c$ provided that $f(c)$ exists, $\lim_{x \to c} f(x)$ exists, and $f(c) = \lim_{x \to c} f(x)$.
   - Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous at all points in their domains.
   - Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes.
7. Determine the applicability of important calculus theorems using continuity.
   - Continuity is an essential condition for theorems such as the Intermediate Value Theorem, the Extreme Value Theorem, and the Mean Value Theorem.

New Jersey Student Learning Standards

- HSN.RN.A: Extend the properties of exponents to rational exponents.
- HSN.RN.B: Use properties of rational and irrational numbers.
- HSN.Q.A: Reason quantitatively and use units to solve problems.
- HSN.VM.A: Represent and model with vector quantities.
- HSN.VM.B: Perform operations on vectors
- HSA.SSE.A: Interpret the structure of expressions
- HSA.SSE.B: Write expressions in equivalent forms to solve problems
- HSA.APR.A: Perform arithmetic operations on polynomials
- HSA.APR.B: Understand the relationship between zeros and factors of polynomials
- HSA.APR.C: Use polynomial identities to solve problems
- HSA.APR.D: Rewrite rational expressions
- HSA.CED.A: Create equations that describe numbers or relationships
- HSA.REI.A: Understand solving equations as a process of reasoning and explain the reasoning.
- HSA.REI.B: Solve equations and inequalities in one variable
- HSA.REI.C: Solve systems of equations
- HSA.REI.D: Represent and solve equations and inequalities graphically
- HSF.IF.A: Understand the concept of a function and use function notation
- HSF.IF.B: Interpret functions that arise in applications in terms of the context
- HSF.IF.C: Analyze functions using different representations
- HSF.BF.A: Build a function that models a relationship between two quantities.
- HSF.BF.B: Build new functions from existing functions

- HSF.LE.A: Construct and compare linear, quadratic and exponential models and solve problems
- HSF.LE.B: Interpret expressions for functions in terms of the situation they model
- HSF.TF.A: Extend the domain of trigonometric functions using the unit circle
- HSF.TF.B: Model periodic phenomena with trigonometric functions
- HSF.TF.C: Prove and apply trigonometric identities
- HSG.SRT.A: Understand similarity in terms of similarity transformations
- HSG.SRT.B: Prove theorems involving similarity
- HSG.SRT.C: Define trigonometric ratios and solve problems involving right triangles
- HSG.SRT.D: Apply trigonometry to general triangles
- HSG.C.A: Understand and apply theorems about circles
- HSG.C.B: Find arc lengths and areas of sectors of circles
- HSG.GPE.B: Use coordinates to prove simple geometric theorems algebraically
- HSG.GMD.B Visualize relationships between two-dimensional and three-dimensional objects
- HSG.MG.A: Apply geometric concepts in modeling situations

Suggested Activities/Modifications

Differentiation strategies may include, but are not limited to, learning centers and cooperative learning activities in either heterogeneous or homogeneous groups, depending on the learning objectives and the number of students that need further support and scaffolding, versus those needing more challenge and enrichment. Modifications may also be made as they relate to the special needs of students in accordance with their Individualized Education Programs (IEPs) or 504 plans, or English Language Learners (ELL). These may include, but are not limited to, extended time, copies of class notes, refocusing strategies, preferred seating, study guides, and/or suggestions from special education or ELL teachers.

New Jersey Student Learning Standards – Technology

- 8.1.12.A – F: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

Career Readiness Practices

- CRP1: Act as a responsible and contributing citizen and employee.
- CRP2: Apply appropriate academic and technical skills.
- CRP4: Communicate clearly and effectively and with reason.
- CRP6: Demonstrate creativity and innovation.
- CRP7: Employ valid and reliable research strategies.
- CRP8: Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9: Model integrity, ethical leadership and effective management.
- CRP10: Plan education and career paths aligned to personal goals.
- CRP11: Use technology to enhance productivity.

- CRP12: Work productively in teams while using cultural global competence.

NJSLS Standard 9.2 - Career Awareness, Exploration, and Preparation

- 9.2.12.C.1 - Review career goals and determine steps necessary for attainment.
- 9.2.12.C.3 - Identify transferable career skills and design alternate career plans.

NJSLS Standard 9.3 Career and Technical Education

- 9.3.ST.2 - Use technology to acquire, manipulate, analyze and report data.
- 9.3.ST.5 - Demonstrate an understanding of the breadth of career opportunities and means to those opportunities in each of the Science, Technology, Engineering & Mathematics Career Pathways.
- 9.3.ST-SM.3 Analyze the impact that science and mathematics has on society.
- 9.3.ST-SM.4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.

UNIT 2: DERIVATIVES

Enduring Understanding

Students will understand that...

1. The derivative is a rate of change at a moment in time.
2. The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.
3. A function's derivative, which is itself a function, can be used to understand the behavior of the function.
4. The derivative has multiple representations and applications including those that involve instantaneous rates of change.
5. Differentiability and continuity are related.

Essential Question(s)

1. What is the connection between the limit and derivative of the function at a given point?
2. How does the concept of the derivative show how a function is changing?
3. How are derivatives used to locate extrema and points of inflection for a function?
4. How do we solve differential equations that have two variables?
5. What are the properties needed to graph a derivative $f'(x)$ from the original function f?
6. How are one-sided derivatives related to a function's overall derivative being defined?
7. What are the cases where $f'(x)$ fails to exist? Why?
8. What is the relationship between differentiability and continuity? Is the relationship reversible?

Learning Objectives

Students will be able to...

1. Identify the derivative of a function as the limit of a difference quotient.

- The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.
- The instantaneous rate of change of a function at a point can be expressed by or $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ provided that the limit exists. These are common forms of the definition of the derivative and are denoted $f'(a)$.
- For $y=f(x)$ notations for the derivative include $\frac{dy}{dx}, f'(x)$, and y'.
- The derivative of f is the function whose value at x is $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ provided this limit exists.
- The derivative can be represented graphically, numerically, analytically, and verbally

2. Estimate the derivative.
   - The derivative at a point can be estimated from information given in tables or graphs
3. Calculate derivatives.
   - Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
   - Specific rules can be used to calculate derivatives for classes of functions, including polynomial, rational, power, exponential, logarithmic, trigonometric, and inverse trigonometric.
   - Sums differences products, and quotients of functions can be differentiated using derivative rules.
   - The chain rule provides a way to differentiate composite functions
   - The chain rule is the basis for implicit differentiation.
   - The chain rule can be used to find the derivative of an inverse function, provided that derivative of that function exists.
   - Methods for calculating derivatives of real-valued functions can be extended to vector-valued functions, parametric functions, and functions in polar coordinates.
4. Determine higher order derivatives.
   - Differentiating f' produces the second derivative f'', provided the derivative of f' exists, repeating this process produces higher order derivatives of f.
   - Higher order derivatives are represented with a variety of notations. For $y=f(x)$ notations for the second derivative include $\frac{d^2y}{dx^2}, f''(x)$ and y''. Higher order derivatives can be denoted $\frac{d^ny}{dx^n}$ or $f^{(n)}(x)$.
5. Use derivatives to analyze properties of a function.

- First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.
- Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.
- Key features of the graphs of f, f', and f'' are related to one another

6. Recognize the connection between differentiability and continuity.
   - A continuous function may fail to be differentiable at a point in its domain
   - If a function is differentiable at a point, then it is continuous at that point
7. Interpret the meaning of a derivative within a problem.
   - The unit for $f'(x)$ is the unit for f divided by the unit for x.
   - The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable
8. Solve problems involving the slope of a tangent line.
   - The derivative at a point is the slope of the line tangent to a graph at that point on the graph.
   - The tangent line is the graph of a locally linear approximation of the function near the point of tangency.
9. Solve problems involving related rates, optimization, rectilinear motion, and planar motion
   - The derivative can be used to solve rectilinear motion problems involving position, speed, velocity, and acceleration.
   - The derivative can be used to solve related rates problems, that is, finding a rate at which one quantity is changing by relating it to other quantities whose rates of change are known.
   - The derivative can be used to solve optimization problems, that is, finding a maximum or minimum value of a function over a given interval.
   - Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along curves given by parametric and vector-valued functions.
10. Solve problems involving rates of change in applied contexts.
11. Verify solutions to differential equations.
    - Solutions to differential equations are functions or families of functions
    - Derivatives can be used to verify that a function is a solution to a given differential equation.
12. Estimate solutions to differential equations
    - Slope fields provide visual clues to the behavior of solutions to first order differential equations
    - For differential equations, Euler's method provides a procedure for approximating a solution or a point on a solution curve.
13. Apply the Mean Value Theorem to describe the behavior of a function over an interval.

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- 9.2.12.C.1 - Review career goals and determine steps necessary for attainment.
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- 9.3.ST-SM.4 Apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.

UNIT 3: INTEGRALS AND THE FUNDAMENTAL THEOREM OF CALCULUS

Enduring Understanding

Students will understand that...

1. Antidifferentiation is the inverse process of differentiation.
2. The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
3. The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
4. The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.
5. Antidifferentation is an underlying concept involved in solving separable differential equations. Solving separable differential equations involves determine a function or relation given its rate of change.
6. Advanced integration methods can be used when *u*-substitution fails

Essential Question(s)

1. What is the fundamental theorem of calculus and its significance?
2. How does the concept of integration help us to find the area under a curve?
3. What are the similarities and differences among the disk and washer methods of integration?
4. How accurately does the rectangular approximation method calculate the area under a curve?
5. What methods can be used to find the integral values of discontinuous functions?
6. How are differential equations used to construct slope fields?

Learning Objectives

Students will be able to:

1. Recognize antiderivatives of basic functions.
   - An antiderivative of a function f is a function g whose derivative is f.
   - Differentiation rules provide the foundation for finding antiderivatives.
2. Interpret the definite integral as the limit of a Riemann sum, as well as express the limit of a Riemann sum in integral notation.
   - A Riemann sum, which requires a partition of an interval I, is the sum of products, each of which is the value of the function at a point in a subinterval multiplied by the length of that subinterval of the partition.
   - The definite integral of a continuous function f over the interval $[a, b]$, denoted by $\int_a^b f(x)\, dx$, is the limit of Riemann sums as the widths of the subintervals approach 0
   - The information in a definite integral can be translated into the limit of a related Riemann sum, and the limit of a Riemann sum can be written as a definite integral.
3. Approximate a definite integral.
   - Definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally.

- Definite integrals can be approximated using a left Riemann sum, a right Riemann sum, a midpoint Riemann sum, or a trapezoidal sum; approximations can be computed using either uniform or nonuniform partitions.

7. Calculate the definite integral using areas and properties of definite integrals.
   - In some cases, a definite integral can be evaluated by using geometry and the connection between the definite integral and area
   - Properties of definite integrals include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
   - The definition of the definite integral may be extended to functions with removable or jump discontinuities
8. Evaluate an improper integral or show that an improper integral diverges.
   - An improper integral is an integral that has one or both limits infinite or has an integrand that is unbounded in the interval of integration.
   - Improper integrals can be determined using limits of definite integrals.
9. Analyze functions defined by an integral.
   - The definite integral can be used to define new functions; for example, $f(x) = \int_0^x e^{-t^2}\,dt$.
   - If f is a continuous function on the interval $[a, b]$, then $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$, where x is between a and b.
   - Graphical, numerical, analytical, and verbal representations of a function f provide information about the function g defined as $g(x) = \int_a^x f(t)\,dt$.
10. Calculate antiderivatives, and evaluate definite integrals.
    - The function defined by $F(x) = \int_a^x f(t)\,dt$ is an antiderivative of f.
    - If f is a continuous function on the interval $[a, b]$, and F is an antiderivative of f, then $\int_a^b f(x)\,dx = F(b) - F(a)$.
    - The notation $\int f(x)\,dx = F(x) + C$ means that $F'(x) = f(x)$, and $\int f(x)\,dx$ is called an indefinite integral of the function *f*.
    - Many functions do not have closed form antiderivatives.
    - Techniques for finding antiderivatives include algebraic manipulation such as long division and completing the square, substitution of variables, integration by parts, and nonrepeating linear partial fractions.
11. Interpret the meaning of a definite integral within a problem.
    - A function defined as an integral represents an accumulation of a rate of change
    - The definite integral of the rate of change of a quantity over an interval gives the net change of that quantity over that interval
    - The limit of an approximating Riemann sum can be interpreted as a definite integral.
12. Apply definite integrals to problems involving the average value of a function, which for the function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.
13. Apply definite integrals to problems involving motion.

- For a particle in rectilinear motion over an interval of time, the definite integral of velocity represents the particle's displacement over the interval of time, and the definite integral of speed represents the particle's total distance traveled over the interval of time.

14. Apply definite integral to problems involving area, and volume
    - Areas of certain regions in the plane can be calculated with definite integrals.
    - Volumes of solids with known cross-sections including discs and washers, can be calculated with definite integrals.
15. Use the definite integral to solve problems in various contexts, such as to express information about accumulation and net change in many applied contexts.
16. Analyze differential equations to obtain general and specific solutions.
    - Antidifferentiation can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay, and logistic growth.
    - Some differential equations can be solved by separation of variables
    - Solutions to differential equations may be subject to domain restrictions
    - The function F defined by $F(x) = c + \int_a^x f(t)\ dt$ is a general solution to the differential equation $\frac{dy}{dx} = f(x)$, and $F(x) = y_0 + \int_a^x f(t)\ dt$ is a particular solution to the differential equation $\frac{dy}{dx} = f(x)$ satisfying $F(a) = y_0$.
17. Interpret, create, and solve differential equations from problems in context.
    - The model for exponential growth and decay that arises from the statement "The rate of change of a quantity is proportional to the size of the quantity" is $\frac{dy}{dt} = ky$.
    - The model for logistic growth that arises from the statement "The rate of change of a quantity is jointly proportionally to the size of the quantity, and the difference between the quantity and the carrying capacity is $\frac{dy}{dt} = ky(a - y)$."

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- HSA.APR.B: Understand the relationship between zeros and factors of polynomials
- HSA.APR.C: Use polynomial identities to solve problems
- HSA.APR.D: Rewrite rational expressions
- HSA.CED.A: Create equations that describe numbers or relationships

- HSA.REI.A: Understand solving equations as a process of reasoning and explain the reasoning.
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- HSG.SRT.B: Prove theorems involving similarity
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- HSG.C.B: Find arc lengths and areas of sectors of circles
- HSG.GPE.B: Use coordinates to prove simple geometric theorems algebraically
- HSG.GMD.B Visualize relationships between two-dimensional and three-dimensional objects
- HSG.MG.A: Apply geometric concepts in modeling situations

<u>Suggested Activities/Modifications</u>
Differentiation strategies may include, but are not limited to, learning centers and cooperative learning activities in either heterogeneous or homogeneous groups, depending on the learning objectives and the number of students that need further support and scaffolding, versus those needing more challenge and enrichment. Modifications may also be made as they relate to the special needs of students in accordance with their Individualized Education Programs (IEPs) or 504 plans, or English Language Learners (ELL). These may include, but are not limited to, extended time, copies of class notes, refocusing strategies, preferred seating, study guides, and/or suggestions from special education or ELL teachers.

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UNIT 4: PLANE CURVES, PARAMETRIC EQUATIONS, AND POLAR CURVES

Enduring Understanding

Students will understand that...

1. Arc lengths are computed as a natural extension of the distance formula.
2. Motion in the plane can be described using parametric equations
3. Polar coordinates can be used to describe curves in the plane.
4. Calculus can be used to find the areas of regions bounded by polar curves.

Essential Question(s)

1. How are arc lengths for non-linear functions computed?
2. How can we convert between polar and rectangular coordinates?
3. How do we describe the slope of a polar curve?
4. How do we describe the motion of a particle moving along a polar curve?
5. How do we determine the area of a region bounded by a polar curve?

Learning Objectives

Students will be able to:

1. Calculate derivatives.
   - For a set of parametric equations where $f(t) = \langle x(t), y(t) \rangle$, then $f'(t) = \langle x'(t), y'(t) \rangle$ and rate of change of the function overall would be $\frac{dy}{dx} = \left(\frac{dy}{dt}\right) \Big/ \left(\frac{dx}{dt}\right) = \frac{dy}{dt} \cdot \frac{dt}{dx}$, provided $\frac{dx}{dt} \neq 0$.
   - For a curve given by a polar equation $r = f(\theta)$, derivatives of *r*, *x*, and *y* with respect to θ and the first and second derivatives of *y* with respect to *x* can provide information about the curve.
2. Determine higher order derivatives
   - For a set of parametric equations where $f(t) = \langle x(t), y(t) \rangle$, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$, provided $\frac{dx}{dt} \neq 0$.
3. Use derivatives to analyze properties of a function.
   - The relationship between slope and tangent can be described in terms of a third independent variable.
   - The slope of a parametric or polar curve is dy/dx.
4. Apply definite integrals to problems involving motion.
   - The definite integral can be used to determine displacement, distance, and position moving along a curve given by the parametric or vector-values functions.
5. Apply definite integral to problems involving area and length of a curve.
   - Bounded by polar curves can be calculated with definite integrals.
   - The length of a planar curve defined by a function or by a parametrically defined curve can be calculated using a definite integral.

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UNIT 5: POLYNOMIAL APPROXIMATIONS AND SERIES
<u>Enduring Understanding</u>
Students will understand that...
1. The sum of an infinite number of real numbers may converge.
2. Not all series have a finite sum. It is often only possible to approximate the sum of an infinite series and to find an associated error bound.
3. A function can be represented by an associated power series over the interval of convergence for the power series.
4. Higher degree polynomials can be used to approximate a variety of functions.
5. All approximations have some degree of error associated with them. As more terms are added to the polynomial, the approximations become more accurate.

<u>Essential Question(s)</u>
1. When does a series converge?
2. How accurate is an estimated from a truncated series?
3. How is a Taylor polynomial constructed, and what is the significance of the point about which it’s centered?
4. When is it appropriate to approximate a result, rather than calculate it directly?
5. What is an acceptable level of error?

Learning Objectives

Students will be able to:

1. Determine whether a series converges or diverges.
   - The nth partial sum is defined as the sum of the first *n* terms of a sequence.
   - An infinite series of numbers converges to a real number *S* (or has sum *S*), if an only if the limit of its sequence of partial sums exists and equals *S*.
   - Common series of numbers include geometric series, the harmonic series, and *p*-series.
   - A series may be absolutely convergent, conditionally convergent, or divergent.
   - If a series converges absolutely, then it converges.
   - In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers converges or diverges are the nth term test, the geometric series test, the direct comparison test, the limit comparison test, the integral test, the ratio test, the root test, and the alternating series test.
2. Determine or estimate the sum of a series.
   - If *a* is a real number and *r* is a real number such that $|r| < 1$, then the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$.
   - If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series.
   - If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the same value.
3. Construct and use Taylor polynomials.
   - The coefficient of the *n*th-degree term in a Taylor polynomial centered at *x* = *a* for the function *f* is $\frac{f^{(n)}(a)}{n!}$.
   - Taylor polynomials for a function *f* centered at *x* = a can be used to approximate function values of *f* near *x* = *a*.
   - In many cases, as the degree of a Taylor polynomial increases, the *n*th-degree polynomial will converge to the original function over some interval.
   - The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function.
   - In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation the function.
4. Write a power series representing a given function.
   - A power series is a series of the form $\sum_{n=0}^{\infty} a_n (x-r)^n$ where *n* is a non-negative integer, $\{a_n\}$ is a sequence of real number, and *r* is a real number.

- The Maclaurin series for $\sin(x)$, $\cos(x)$, and e^x provide the foundation for constructing the Maclaurin series for other functions.
- The Maclaurin series for $\frac{1}{1-x}$ is a geometric series.
- A Taylor polynomial for $f(x)$ is a partial sum of the Taylor series for $f(x)$.
- A power series for a given function can be derived by various methods (e.g. algebraic processes, substitutions, using properties of geometric series, and operations on known series such as term0by-term integration or term-by-term differentiation).

5. Determine the radius and interval of convergence of a power series.
    - If a power series converges, it either converges at a single point, or has an interval of convergence.
    - The ratio test can be used to determine the radius of convergence of a power series.
    - If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval.
    - The radius of convergence of a power series obtained by term-by-term differentiation or term-by-term integration is the same as the radius of convergence of the original power series.

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V. **Course Materials** (included, but not limited to)

- **Textbook**
  Title: *Calculus: Graphical, Numerical, Algebraic, AP Edition*
  Author: Ross Finney, Franklin Demana, Bert Waits, Daniel Kennedy
  Year: 2007

- **Supplemental Resources**
  Title: Preparing for the 2015 Calculus (BC) Exam
  Author: George Best, Richard Lux
  Year: 2014

  Title: Be Prepared for the AP Calculus Exam, 2nd Edition
  Author: Mark Howell, Martha Montgomery
  Year: 2011

  Title: Barron's AP Calculus, 9th Edition
  Author: Shirley Hockett, David Bock
  Year: 2008

  Title: Preparing for the Calculus AP* Exam (accompany classroom textbook)
  Author: Ray Barton, John Brunsting, John Diehl, Greg Hill, Karyl Tyler, Steven Wilson
  Year: 2012

- **Technology:**
  TI-Nspire Graphing Calculator
  Google Applications

VI. **Assessments** (included, but not limited to)

- Do Now Problems
- Quizzes
- Tests
- Classwork
- Homework
- Midterm Exam
- Final Project
- AP Calculus Review Binder
- AP Calculus Exam Practice

VII. Cross Curricular Aspects

- Use of Calculator (Sciences)

- Applications of Differentiation: Motion, Related Rates, Optimization, etc. (Science and Business)
- Applications of Integration: Slope Fields, Volume, etc. (Arts)
- Final Alternative Assessment (Arts and Humanities)
- Problem-Solution Writing and Discussion: exposition, problem statement, road map, solution, and extension. (Humanities)